21st ANNUAL SYSTEM-WIDE STUDENT MATH CONTEST QUESIONS: 2011 CONNECTICUT COMMUNITY COLLEGES
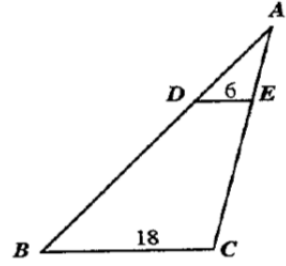
Each of the following problems, 1 through 6, is worth one (1) point.

1. Find all of the 2 digit prime numbers, such that neither of the digits is prime.

2. The sum of 2 integers is 28, and their product is 192. What are the 2 integers?

3. The numbers 3, a, 10, b form an arithmetic sequence. Compute the value of $a + b$.

4. Given the following system of equations: $y = -x + 6$, and $y = \frac{x}{3} + C$, with the solution lying in Quadrant 1, then find all possible values of C.

5. If $x + y = 0$, and x is not $= 0$, then what is the value of $\dfrac{x^{2011}}{y^{2011}}$?

6. Robby Cano got 4 less than twice as many hits as Kevin Youkilis. Altogether, they got 11 hits. How many hits did Cano get?

Each of the following problems, 7 through 14, is worth two (2) points.

7. Find the dimensions of a right triangle, with integral values of the sides, such that the area and the perimeter have the same numerical value.

8. A Pythagorean Quadruple is a set of positive integers a, b, c and d, such that $a^2 + b^2 + c^2 = d^2$. If 2, 9 and 11 are three of these 4 integers that form a quadruple, then find the 4th integer.

9. A banquet hall has a capacity of 400 people, including both diners and servers. If 1 server is needed for every 12 diners, then what is the maximum number of diners who can be served?

10. What is the length of the diagonal of an isosceles trapezoid, with side lengths of 7, 8, 8, and 15?

11. How many different real numbers satisfy the equation $(x^2 + 4x - 2)^2 = (5x^2 - 1)^2$?

12. Solve for x: $4^{(1 - 4x)} = 8^{(3x - 1)}$

13. In triangle ABC, $\overline{DE}$ is parallel to $\overline{BC}$, $DE = 6$, $BC = 18$, and $AD = 12$. If the perimeter of triangle ABC = 81, then what is the length of $\overline{EC}$?

14. Find the average of all multiples of 7, between 7 and 777, inclusive.

Each of the following problems, 15 through 20, is worth three (3) points.

15. In a particular solution weighing 200 grams, 99 % of its weight is water. After a while, some water evaporates from the solution, but no other liquid loses volume. The remaining solution is now 98% water. How many grams of water evaporated?

16. A rectangle with side lengths of a and b has a diagonal length of 12 units. If the perimeter of the rectangle is the same as the perimeter of a square with area of 76 square units, then find $a \cdot b$.

17. In the following diagram, the digits 2, 4, 5, 6, 8 and 9 can be distributed among the lettered squares in the array so that the sum of the entries in each of the rows and columns is the same number k. What is k?

7	a	b	1
c			d
3	e	f	10

18. Two non-zero numbers, a and b, satisfy $ab = a - b$. What is the possible value of $\frac{a}{b} + \frac{b}{a} - ab$?

19. If x, y and z are positive numbers satisfying
$x + \frac{1}{y} = 4$, $y + \frac{1}{z} = 1$, and $z + \frac{1}{x} = \frac{7}{3}$, then find the value of $x \cdot y \cdot z$.

20. The children in a family comprise both boys and girls. Each boy has as many brothers as sisters, but each girl has half as many sisters as brothers. How many children are in the family?

2011 MATH CONTEST ANSWER KEY

1. 11, 19, 41, 61, 89
2. 16 and 12
3. $a + b = 20$
4. $-2 < C < 6$
5. -1
6. Cano got 6 hits

7. 6-8-10 or 5-12-13
8. The 4th integer is 6
9. 369
10. 13
11. 3 real solutions: $x = 0.5$, or $x = \dfrac{-2 \pm \sqrt{22}}{6}$
12. $x = \dfrac{5}{17}$
13. EC = 18
14. Average = 392

15. 100 grams of water evaporated
16. $ab = 80$
17. $k = 19$
18. 2
19. $xyz = 1$
20. 7 children in the family