The 30th Annual Connecticut Community Colleges Math Contest

Directions for Student Participants

Please read these directions carefully before starting the test!

1. Only students currently enrolled in the community college system are eligible to participate.
2. Do not begin the test until instructed by the test monitor.
3. You have two hours to complete all the questions. Some questions are worth 1 point, some are 2 points, some are 3 points, and some are 4 points.
4. You are allowed to use calculators. No books, notes, or other aids are allowed. You may not share calculators during the test.
5. You will be provided with scrap paper and graph paper, on which you can do all your work.
6. All answers MUST be recorded on the answer sheet provided. Answers must be fully simplified, and exact answers must be given unless otherwise specified.
7. All answers must be complete, legible, and with the proper units or labels (for example: inches, pounds, dollars, miles per hour, etc.) No partial credit is given.
8. Please record all answers with a ball point pen.
9. Please sign the answer sheet and initial the test question sheet with a ball point pen.
10. Please return all test papers to the test monitor before leaving which you can do once you are done).

Sincerely, the Contest Committee

The 30th Annual Miguel Garcia Math Contest

Sponsored by MATYCONN

Spring 2021

One Point Questions:

1. What number gives the same result when it is added to ½ as when it is multiplied by ½ ?

2. Given the function $f(a,b) = \frac{a}{b} + \frac{b}{a} + \frac{1}{ab}$ find $f(2,5)$

3. The sum of Zip's age and Dina's age is 51. The sum of Julio's age and Dina's age is 54. Zip is 7 years old. How old is Julio?

4. Joyce has two identical jars. The first jar is ¾ full of water and contains 300mL of water. The second jar is ¼ full of water. How much water, in mL, does the second jar contain? Include units.

5. David owns a parking lot for vehicles. A vehicle is either a motorcycle with two wheels or a car with four wheels. Today, there are 100 vehicles parked in his parking lot. The total number of wheels in David's parking lot is 326. If David collects $1 from each motorcycle and $2 from each car per day, how much money, in dollars, does David collect today.

6. Find a number with the following properties:
   - It is a prime number larger than 3
   - Two raised to the power equal to this number is three more than a perfect cube

Two Point Questions:

7. A circular track has a radius of $60\ meters$. Ali runs around the circular track at a constant speed of $6\ m/s$. A track in the shape of an equilateral triangle has a side length of x meters. Darius runs around the triangular track at a constant speed of $5\ m/s$. Ali and Darius each complete one lap in exactly the same amount of time. What is the value of x? Include units.

8. What integer satisfies $3 < \frac{24}{a} < 4$?

9. Find all solutions to: $\frac{1}{x^2} - \frac{1}{x} = 2$

10. If $x^2 + xy + y^2 = a$ and $x + y = b$, what is the value of xy in terms of only a and b

11. Nick and Steve collect calculators. If Steve gave Nick two calculators, they would have the same amount. If Nick gave Steve two calculators, Steve would have twice as many as Nick. How many calculators do they have together?

12. Andrea, Barbara, and Claire each own a Lamborghini and a monkey; however, they are each driving someone else's car with someone else's monkey in the passenger seat. The woman driving Barbara's car has Claire's monkey. Who is driving Andrea's car?

13. Find the y-intercept as an ordered pair of the line that passes through point $(9, 2)$ and is parallel to line $x + 3y = 7$

14. Find a four-digit number where the thousands digit is four greater than the hundreds digit, the tens digit is twice the thousands digit, and the ones digit is half of the thousands digit.

Three Point Questions:

15. Solve the problem for the real number x:

$$16^{\frac{15}{x}} = 32^{\frac{4}{3}}$$

16. Consider the following triangle with vertices at $(0, 0)$, $(0, 4)$, and $(6, 0)$. What is the fraction of all integer ordered pairs within the triangle, including the edges, that have an x-coordinate that is less than its y-coordinate?

17. The arithmetic mean of two numbers x and y is found as $\frac{x+y}{2}$
The geometric mean of two numbers x and y is found as $\sqrt{xy}$
Find a pair of positive real numbers whose arithmetic mean is 13 and geometric mean is 12.

18. Find the real value for w to complete the arithmetic sequence: $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{1}{w}$

19. Michelle calculates the average of the following numbers:

$$5, 10, 15, 16, 24, 28, 33, 37$$

Sue removes one of the numbers and calculates the average of the remaining numbers. The average Sue calculates is one less than the average Michelle calculates. Which number did Sue remove?

20. If $a_n = \frac{1}{n} - \frac{1}{n+1}$, Find the sum: $a_1 + a_2 + a_3 + a_4 + a_5$

Four Point Questions:

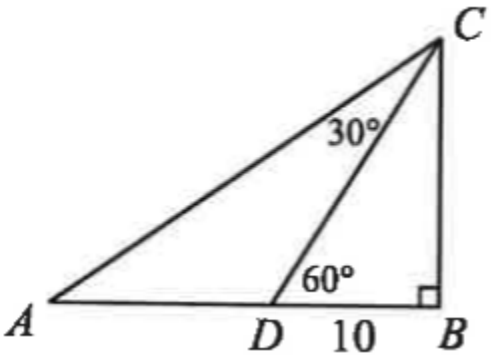
21. Michelle, Paul, and Harry play three different instruments and play shows in three different cities.
    a. Paul watched the drummer in Orlando.
    b. Michelle watched the pianist in Milwaukee.
    c. Harry watched the guitarist in Phoenix.
    d. Paul doesn't play the piano.

    What city does Harry perform in?

22. Find the ratio of the circumference of a circle to the perimeter of a square inscribed in the circle.

23. If $f(x-1) = x^2 - 3x + 5$ what does $f(x+1) =$?

24. Find the length of AD for the following triangle:



25. In the 2009 World Series between the Yankees and the Phillies a total of 59 runs were scored over 6 games. In the first game, the Phillies won by 5 runs. In the second game the Yankees won by 2 runs. In the third and fourth games, the Yankees won by 3 runs each time. In the fifth game, the Phillies won by 2 runs. In the sixth game, the Yankees won by 4 runs. How many total runs did the Yankees score over the 6 games?