

# The 27<sup>th</sup> Annual Miguel Garcia Math Contest

*sponsored by MATYCONN*

**April 2017**

## **One Point Questions:**

1. Find the smallest integer  $y$  which satisfies the inequality:

$$8 < 4y + 4 < 20$$

2. A test consists of 20 questions. For every correct answer, a student earns 7 points. For every wrong answer, a student loses 2 points. For every question the student doesn't answer, the student gets 0 points. Janet got 87 points. How many questions did she not answer?
3. How many three-digit numbers are there whose sum of the digits is 4?
4. What is the smallest positive integer  $n$  such that the fraction:  $\frac{5n+6}{8n+7}$  can be reduced?
5. If  $\frac{1}{x} + \frac{1}{y} = \frac{5}{6}$  and  $xy = 3$ , what is the value of  $x + y$ ?
6. Gary Sanchez hit 3 less than twice as many homeruns as Mookie Betts. Betts also hit 14 fewer homeruns than Sanchez. How many homeruns did Sanchez hit?

### Two Point Questions:

7. Find all values of  $D$ , strictly between 1 and 104 such that  $122 + D$  is a perfect square.
8. Winnie has won 30 out of 50 chess games against her computer. What is the fewest number of consecutive additional games that she needs to win if she wishes to improve her success rate to at least 65%.
9. If  $A$  is 125% of  $B$ , what percent of  $A$  is  $B$ ?
10. Find the value of  $k$ , such that the system  $kx - 3y = 4$  and  $4x - 5y = 7$  has no solution.
11. Find the smallest range of 5 positive integers whose mean is 7, median is 8, and mode is 9.
12. How many unique two letter arrangements can be made from the letters in: "EULER" (Note: that EU and UE are two unique arrangements)
13. If you have socks that are blue, black, and brown; but are too lazy to fold them in matched pairs, then what is the fewest number of singles socks you need to take from your drawer to guarantee you will get two matching pairs.
14. If  $A, B, M,$  and  $N$  are positive integers, and  $\frac{M}{N} = \frac{A}{A-B}$ , What expression is  $x$  in the following equation:  $\frac{A}{B} + \frac{M}{N} = \frac{AM}{x}$

### Three Point Questions:

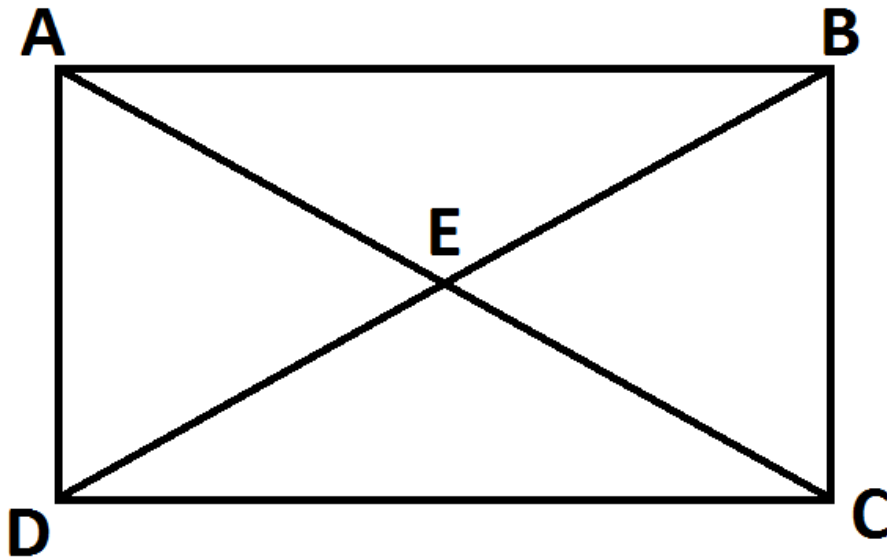
15. To get ready for Easter, I bought 100 jellybeans and divided them among five different bowls in my house. The first and second bowl together contain 48 jelly beans. The second and third bowl together contain 34 jelly beans. The third and fourth bowl together contain 30 jelly beans. The fourth and fifth bowl together contain 48 jelly beans. How many jelly beans are in the bowl with the least amount of jelly beans?
16. Caitlyn left her house at 11am. She averaged 45mph on her trip to the city because of roadwork. She spent 90 minutes in the city and returned home on the same route averaging 60mph. If she arrived home at 6:30pm, then what is the distance from Caitlyn's house to the city rounded to the nearest mile?
17. The operation  $A^{\circ}B$  is defined as  $A \times B + 3$ . What is the positive difference between  $10^{\circ}(11^{\circ}12)$  and  $(10^{\circ}11)^{\circ}12$  ?
18. What is the value of:  $(1 - 2) - (3 - 4) - (5 - 6) - \dots - (99 - 100)$  ?
19. Two numbers  $x$  and  $y$  sum to 23, and their difference is 1. How many unique factors does their product have?
20. Let  $N = p^2 - p + 1$ , where  $p$  is a prime number. What is the smallest value of  $p > 2$  such that  $N$  is a perfect cube?

**Four Point Questions:**

21. For all values of  $x$ ,  $(Ax + 3)(Bx + 5) = 12x^2 + Cx + 15$   
If  $A + B = 8$ , then what are all the possible values of  $C$ ?

22. If  $100 + 101 + 102 + \dots + (n - 1) + n = 2800$ , then what is the value of  $n$ ?

23. In the following diagram, how many different ways can you get from **A** to **E** without repeating any paths?



24. The least common multiple of  $m$  and  $n$  is 140. If  $m = 20$  what are all of the possible values of  $n$ ?

25. In the magical land of MATYCONN, dragons either have six, seven, or eight heads. Dragons with seven heads always lie, but dragons with six or eight heads always tell the truth. One day four dragons meet up. The blue dragon says, "Together we have 28 heads." The red dragon says, "Together we have 26 heads." The green dragon says, "Together we have 27 heads." The yellow dragon says, "Together we have 25 heads." What color are the dragon(s) with six heads?